**Trig Sub Reference Triangle**

![Trig Sub Reference Triangle Diagram]

- $a = \tan \theta$
- $x = a \tan \theta$
- $\sqrt{a^2 + x^2} = \sec \theta$
- $\sqrt{a^2 - x^2} = \sin \theta$
- $\sqrt{x^2 - a^2} = \tan \theta$

**Partial Fraction Decomposition**

Given two functions, $P(x)$ and $Q(x)$, when integrating \( \int \frac{P(x)}{Q(x)} \, dx \), and $P(x)$ is smaller in degree than $Q(x)$, the integral decomposes into:

\[
(ax + b)^k \rightarrow \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \ldots + \frac{A_k}{(ax+b)^k}
\]

\[
(ax^2 + bx + c)^k \rightarrow \frac{A_1x+B_1}{ax^2+bx+c} + \ldots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}
\]

**Trapezoidal Rule**

To approximate $\int_a^b f(x) \, dx$, use

\[
T = \frac{\Delta x}{2} (y_0 + 2y_1 + \ldots + 2y_{n-1} + y_n)
\]

The $y$'s are the values of $f$ at the partition points

- $x_0 = a$, $x_1 = a + \Delta x$, $\ldots$, $x_{n-1} = a + (n-1)\Delta x$
- $x_n = b$, where $\Delta x = \frac{(b-a)}{n}$

**Simpson’s Rule**

To approximate $\int_a^b f(x) \, dx$, use

\[
S = \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n)
\]

The number $n$ is even, and $\Delta x = \frac{(b-a)}{n}$

**Improper Integrals**

**Type I**

1. If $f(x)$ is continuous on $[a, \infty)$, then

\[
\int_a^\infty f(x) \, dx = \lim_{b \to \infty} \int_a^b f(x) \, dx
\]

2. If $f(x)$ is continuous on $(-\infty, b]$, then

\[
\int_{-\infty}^b f(x) \, dx = \lim_{a \to -\infty} \int_a^b f(x) \, dx
\]

3. If $f(x)$ is continuous on $(-\infty, \infty)$, then

\[
\int_{-\infty}^\infty f(x) \, dx = \int_c^c f(x) \, dx + \int_c^\infty f(x) \, dx
\]

**Type II**

1. If $f(x)$ is cont. on $(a, b]$ and discont. at $a$, then

\[
\int_a^b f(x) \, dx = \lim_{c \to a^+} \int_c^b f(x) \, dx
\]

2. If $f(x)$ is cont. on $[a, b)$ and discont. at $b$, then

\[
\int_a^b f(x) \, dx = \lim_{c \to b^-} \int_a^c f(x) \, dx
\]

3. If $f(x)$ is discont. at $c$, where $a < c < b$, and cont. on $[a, c) \cup (c, b]$, then

\[
\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx
\]

*In each case, if the limit is finite, the improper integral **converges** and that the limit is the **value** of the improper integral. If the limit does not exist, the integral **diverges**.*

**Polar Coordinates**

Equations relating Polar and Cartesian

\[
x = r \cos \theta, \quad y = r \sin \theta
\]

\[
\tan \theta = \frac{y}{x}, \quad r^2 = x^2 + y^2
\]
Sequences

The sequence converges to the finite number $L$ if the limit of the sequence

$$\lim_{n \to \infty} a_n = L$$

The sequence diverges to negative or positive infinity

$$\lim_{n \to \infty} a_n = \pm \infty$$

Infinite Series

Given a sequence of numbers $\{a_n\}$, $a_1 + a_2 + \ldots + a_n$, is an infinite series. The number $a_n$ is the $n$th term of the series. The sequence $\sum_{k=1}^{n} a_k$ is the sequence of partial sums of the series. If the sequence of partial sums converges to a limit $L$, we say that the series converges and its sum is $L$. If the sequence of partial sums does not converge to a number, we say it diverges.

Convergence Tests

Comparison Test

Let $\sum a_n$, $\sum c_n$, and $\sum d_n$ be series with nonnegative terms. Suppose, for some number $N$, that

$$d_n \leq a_n \leq c_n$$

for all $n > N$

(A) If $\sum c_n$ converges, then $\sum a_n$ also converges

(B) If $\sum d_n$ diverges, then $\sum a_n$ also diverges

$P$-Series Test

The $P$-Series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, diverges if $p \leq 1$

Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$

If $\lim_{n \to \infty} \frac{a_n}{b_n}$ equals:

1) $c > 0$, then both series either converge or diverge

2) $0$, and $\sum b_n$ converges, then $\sum a_n$ converges

3) $\infty$, and $\sum b_n$ diverges, then $\sum a_n$ diverges

Ratio Test

Let $\sum a_n$ be a series with positive terms, compute

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Root Test

Let $\sum a_n$ be a series with $a_n > 0$ for $n \geq N$

$$\lim_{n \to \infty} \sqrt[n]{a_n} = p$$

Converges if $p < 1$, diverges if $p > 1$ or is infinite, inconclusive if $p = 1$

Alternating Series Test

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 \ldots$$

Converges if the following is satisfied:

all $u_n$’s are positive, the positive $u_n$’s are non-increasing, and $u_n \to 0$