Locking Lasers with Large FM Noise to High Q Cavities

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***Abstract*— We have demonstrated that an external-cavity diode laser can be directly locked to a high Q cavity. Lock can be acquired with a conventional analog servo even though the laser frequency sweeps through the cavity resonance in less than the cavity build-up time. We discuss the distortions of the Pound-Drever-Hall error signals in this regime and experimentally and theoretically show that a servo can acquire lock. We discuss a model of lock acquisition and experimentally show that the capture range is in fact much larger than that predicted from a linear model.**

I. Introduction

Lasers locked to passive cavities with finesses greater than 10^5 generate the most stable frequencies [1]. The demonstration of phase coherent fs combs allow this stability to be transferred to any microwave or optical frequency with many widespread applications [2]. When locking to an ultra-narrow cavity, the free-running laser linewidth can be much greater than the cavity linewidth so that the laser frequency sweeps through the cavity resonance in less time than it takes for light to build up in the cavity [3-5]. This leads to severe distortions of the Pound-Drever-Hall error signal [6] used to control the laser frequency, suggesting that a conventional servo cannot acquire lock [5]. This difficulty has been avoided by locking to a pre-stabilization cavity whose linewidth is smaller, but comparable to the laser linewidth, and a very high Q cavity [1,5]. Here we show that these error signal distortions in fact do not inhibit lock acquisition for narrow cavities and conventional analog servos. Thus, one can either lock directly to a very high Q cavity or use a pre-stabilization cavity that also has a very high Q. A high Q pre-stabilization cavity more strongly attenuates the frequency and amplitude noise of the transmitted light.

We lock an external-cavity diode laser to a cavity with a linewidth of 3.5 kHz using a conventional analog servo [7]. A series of experiments has locked diode lasers to successively narrower cavity resonances, from 50 kHz [8], to 14 kHz [9], to as narrow as 9.5 kHz[10] and 9 kHz [11]. As cavity linewidths become narrower, the distortion threshold goes down quadratically. Our cavity resonance is nearly a factor of three narrower than the previous best, leading to error signal distortions that are 7 times larger for the same laser noise [4]. Here, we intentionally add large laser frequency modulation (>1 MHz) that puts us clearly into this regime for the first time so we can assess lock acquisition.

II. Fast Sweeps Through a Resonance

Fast sweeps through a cavity resonance were previously analyzed in the context of gravity wave detectors. The cavity response to a linear motion the of mirrors was studied in the time domain by solving differential equations [3-5]. In units of the cavity lifetime, the electric field in the cavity is given by [5]

$$\frac{dE_{cav}}{dt} = -(1 - i v_\omega t) E_{cav} + i E_{in}, \qquad (1)$$

where E_{in} is the input electric field, and $v_\omega = \dot{\omega} / \Delta\omega^2$ is the dimensionless frequency velocity for the frequency sweep.

In Fig. 1 we show the Pound-Drever-Hall error signal S_{PDH}[6], which is proportional to the real part of Ecav. For

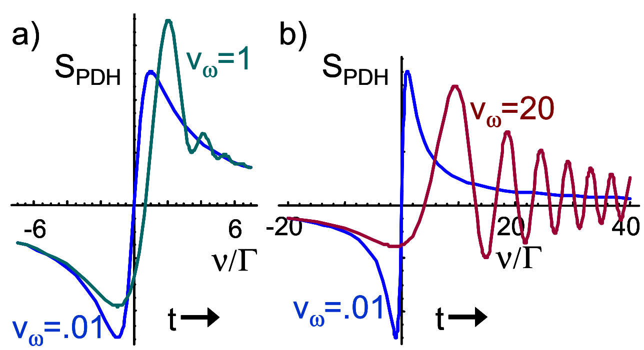

Fig. 1. Pound-Drever-Hall Error signals for a slow sweep through the cavity resonance (v_ω=0.01), a sweep near the distortion threshold of v_ω=1, and a fast sweep (v_ω=20).

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slow sweeps through the cavity resonance, one sees the well-known error signal $S_{PDH}=\beta'\delta\Delta\omega/(\delta^2+\Delta\omega^2)$. As the frequency velocity approaches 1, a fast beat occurs between the sweeping incident light and the field stored in the cavity. This effect is known as Cady clicking in piezoelectric resonators where an audible beat can often be heard [12]. For v_ω=20, the error signal distortions are so large that the error signal often has the wrong sign. This suggests that lock cannot be acquired when $v_\omega \gg 1$ [3-5]. In gravity wave detectors, the lock can be acquired with a fast real-time digital analysis of the error signal distortions. From this, a mirror velocity is deduced and the appropriate force can be applied to the mirror to nearly stop the mirror's motion and then bring the cavity into resonance [3,4,13].

III. Theoretical Model of the Pound-Drever-Hall Error Signal

Here we expand on our theoretical model presented in [7]. Our analysis particularly treats the case of large FM laser noise for the first time in the frequency domain. To analyze the lock acquisition, we treat our real laser as an ideal laser to which frequency noise is added [14]. We decompose the laser frequency noise into its Fourier components at frequency Ω and then treat each frequency individually. If a small range of frequencies dominate in any particular frequency band, this is appropriate. As we note below in the model of lock acquisition, the response is non-linear and more work is needed to fully understand the non-linear dynamics during lock acquisition. The 780 nm light from our laser passes through a 2 m optical fiber and then an electro-optic phase modulator (EOM) which produces frequency sidebands. This light reflects from the cavity and the light that is near resonance is phase shifted and partially transmitted [6]. The electric field of this reflected light is:

$$E=\sum_{m,n}\frac{\delta+m\omega_{EO}+n\Omega}{\delta+m\omega_{EO}+n\Omega+i\Delta\omega} \times J_m(\beta')J_n(\beta)e^{-i(\omega+m\omega_{EO}+n\Omega)t} \qquad (2)$$

where δ is the detuning from the cavity resonance, $\Omega=2\pi f_n$ and β describe the laser frequency noise $f(t)=f_{fm}\cos(\Omega t)$ with $\beta=f_{fm}/f_n$, the cavity linewidth is $\Delta\omega=2\pi\Delta\nu$, and ω_{EO} and β' are the phase modulation frequency and depth of the EOM with $\omega_{EO} \gg \Delta\omega$ [6]. We take β' to be small, the mirrors to be highly reflective, and all frequencies much less than the free-spectral-range [15].

The Pound-Drever-Hall error signal S_{PDH} comes from mixing the photodiode current, proportional to E^*E, with $\sin(\omega_{EO}t)$, and then low-pass filtering [6]. Since ω_{EO} is much larger than $\Delta\omega$ and all f_{fm}, we only need to consider the first order sidebands of ω_{EO}; that is m=0, ±1 in Eq. 1. We take the laser frequency to be bounded around and centered on the cavity resonance (δ=0) so that any slowly varying laser frequency noise is included in low frequency Ω's. This leads to [7]:

$$S_{PDH}=2\beta'\sum_{p,k=0}^{\infty}J_{k-p}(\beta)J_{k+p+1}(\beta)(2k+1)\Delta\omega\Omega \times\frac{\left[(k-p)(k+p+1)\Omega^2+\Delta\omega^2\right]\cos\left[(2p+1)\Omega t\right]+(2p+1)\Delta\omega\Omega\sin\left[(2p+1)\Omega t\right]}{\left[(k-p)^2\Omega^2+\Delta\omega^2\right]\left[(k+p+1)^2\Omega^2+\Delta\omega^2\right]} \qquad (3)$$

This gives S_{PDH} for all noise depths of all noise frequencies, provided that the frequency excursions do not approach ω_{EO} or an adjacent cavity resonance. Eq. 3 accurately represents our measured S_{PDH}, including the Cady clicking for frequency velocities v_ω as large as 100 [7]. The high frequency beats arise from the p>0 odd harmonics of Ω in Eq. 3.

In the usual operating regime after lock is acquired, the laser has small fm noise. In this limit, Eq. 3 reproduces the expected linear and sinusoidal response for sinusoidal frequency modulation, $f(t)=f_{fm}\cos(\Omega t)$.

$$S_{PDH}=\beta' f_{fm}\frac{\Delta\nu\cos(\Omega t)+f_n\sin(\Omega t)}{f_n^{\,2}+\Delta\nu^2} \qquad (4)$$

This response mimics the behavior of a low pass filter with a pole at the cavity linewidth $\Delta\nu$. Low frequencies have a small phase shift and a frequency independent response. High frequencies have a phase lag of $\pi/2$ with the response falling as $1/f_n$ [6].

IV. Pound-Drever-Hall Error Signal Response at the Frequency of the Noise

Can S_{PDH} be used to acquire lock when there are large distortions in the error signal as in Fig. 1? To understand this, we measure and calculate the magnitude and the phase of S_{PDH} at the frequency of the noise Ω [7]. This is the p=0 term in Eq. 3; we label it $S_{PDH,0}$.

We show the magnitude and phase of $S_{PDH,0}$ in Fig. 2 as a function of the amplitude of the frequency noise, f_{fm} or β. There are four different behaviors which we now describe. After understanding the behaviors, in the next section we discuss a simple model of the lock capture range.

A. *Low frequency, $f_n<\Delta\nu$*

For small noise amplitude f_{fm}, $S_{PDH,0}$ is given by Eq. 4. It linearly increases with f_{fm} and has a small phase lag. The phase modulation index for low frequency noise can be very large because $\beta=f_{fm}/f_n$, even when the noise amplitude f_{fm} is less than the cavity linewidth. Here it is more physical to think of the laser frequency sweeping back and forth over the cavity resonance instead of a series of frequency sidebands, as in Eq. 2. $S_{PDH,0}$ traces out the DC response, $S_{PDH}=\beta'\delta\Delta\omega/(\delta^2+\Delta\omega^2)$, as the laser sweeps slowly back and forth over the resonance. As the noise amplitude f_{fm} increases up to the cavity linewidth, $S_{PDH,0}$ increases. When the noise amplitude f_{fm} is greater than the cavity linewidth $\Delta\nu$, the response falls due to the non-linearity of the DC

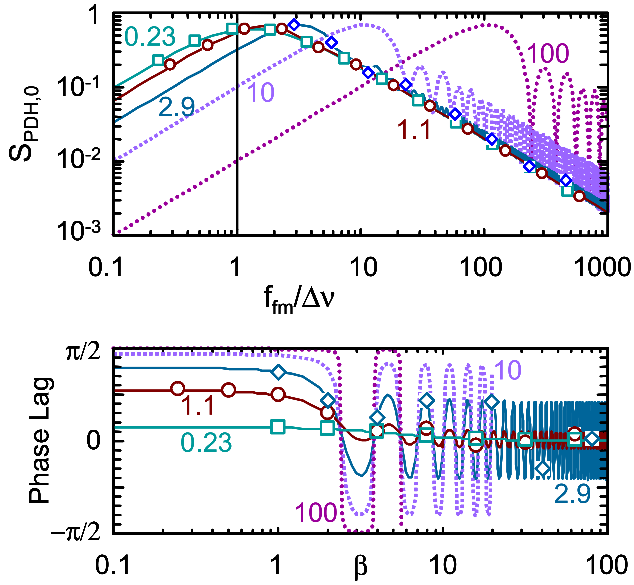

Fig. 2. The magnitude (a) and phase (b) of SPDH at the noise frequency $\Omega=2\pi f_n$ as a function of noise depth f_{fm} and $\beta=f_{fm}/f_n$ for $f_n/\Delta\nu$= 0.23, 1.15, 2.9, 10, and 100. Data points are squares (circles, diamonds) for $f_n/\Delta\nu$=0.23 (1.15, 2.9). The response falls for large fm depth at all f_n. For $f_n>\Delta\nu$, the phase oscillates between a phase lag and a phase lead for $\beta>2$.

response. For large f_{fm}, $S_{PDH,0}$ decreases as $1/f_{fm}$. For large β (whether $f_{fm}<\Delta\nu$ or $f_{fm}\geq\Delta\nu$), $S_{PDH,0}$ reduces to:

$$S_{PDH,0} = 4\beta' \sum_{k=0}^{\infty} \frac{k f_n J_k(\beta) J_{k+1}(\beta)}{k^2 f_n^{\,2} + \Delta\nu^2} \Delta\nu \cos(\Omega t) \quad (5)$$

For large β the product of the Bessel functions sums to zero for small k and is large near $k\approx\beta_n$, corresponding to noise sidebands near the turning points of the frequency sweep. This in-phase component can therefore be easily derived by extracting the amplitude of cos(Ωt) term in the DC frequency modulated response. This gives:

$$S_{PDH,0} = 2\beta' \frac{\Delta\nu}{f_{fm}} \left(1 - \frac{\Delta\nu}{\sqrt{f_{fm}^{\,2} + \Delta\nu^2}} \right) \cos(\Omega t) \quad (6)$$

If $f_{fm}<<\Delta\nu$, then $S_{PDH,0}=\beta' f_{fm}/\Delta\nu\cos(\Omega t)$, increasing linearly with the noise amplitude f_{fm}. If $f_{fm}>>\Delta\nu$, Eqs. 5 and 6 reduce to

$$S_{PDH,0} = 2\beta' \frac{\Delta\nu}{f_{fm}} \cos(\Omega t) \quad (7)$$

and the response falls as $1/f_{fm}$. This limit for the in-phase component is obvious from the amplitude of cos(Ωt) in the DC frequency modulated response. It is interesting that the phase lag for f_n=0.23 Δν in Fib. 2b goes from a small non-zero phase lag, for small f_{fm}, towards zero for large f_{fm}.

As the amplitude of the noise becomes very large, eventually the frequency sweeps through the resonance in less than a cavity lifetime, with a frequency velocity $v_\omega=f_{fm}f_n/\Delta\nu^2$. It is striking that, because $S_{PDH,0}$ in Eqs. 5-7 is dominated by contributions near the turning points, it is entirely independent of v_ω! Thus, sweeping through the cavity resonance in less than the build-up time does not change the large f_{fm} behavior of $S_{PDH,0}$! (It does of course affect the odd harmonics of Ω as in Eq. 2.) Because the magnitude falls slowly for large f_{fm} and the phase shift is small for all f_{fm}, a servo can use $S_{PDH,0}$ to acquire lock when $f_n<\Delta\nu$.

B. *High frequency, $f_n>\Delta\nu$*

There are three distinct behaviors for high noise frequencies.

1) In Fig. 2a, for small noise amplitudes f_{fm}, the response is proportional to the amplitude and, again, for large noise amplitudes, the magnitude falls, but with an oscillation between a maximum and a non-zero minimum. For $f_{fm}<f_n$ (β<1), the amplitude of $S_{PDH,0}$ increases linearly with f_{fm} with a phase lag approaching π/2, as described by Eq. (4). The dominant contribution to $S_{PDH,0}$ in this regime is from the carrier and the first noise sideband.

2) When $f_{fm}>f_n$ (β>1), the magnitude of $S_{PDH,0}$ oscillates with a maximum magnitude of $S_{PDH,0}=2\beta' J_0(\beta)J_1(\beta)\sin(\Omega t)$. This too is obviously dominated by the carrier and the first noise sideband. This scales as $1/\beta(=f_n/f_{fm})$ and goes through zero once for every increase of β by π/2. Therefore the phase of $S_{PDH,0}$ regularly switches between a phase lag of nearly π/2 and a phase lead of nearly π/2, as in Fig. 2b.

3) The minimum magnitude of $S_{PDH,0}$ is the same as Eq. (6). Just as above for $f_n<\Delta\nu$, the dominant contribution is from noise sidebands near the turning points of the frequency sweep. Therefore $S_{PDH,0}$ is always non-zero and has zero phase lag for these β's.

Since the phase lag never exceeds ±π/2, $S_{PDH,0}$ also allows the servo to capture lock for $f_n>\Delta\nu$.

V. Lock Capture Range

Because $S_{PDH,0}$ falls for large noise amplitudes, it is interesting to ask how large is the lock capture range. Eq. (6) gives the magnitude of $S_{PDH,0}$ for large fm at low frequencies and, for high frequency noise, it gives the minimum magnitude of $S_{PDH,0}$. For high frequency noise, one should consider the minimum magnitude because, as the servo attacks the noise in this model, the magnitude of the remaining noise at some point will have a β that gives the minimum magnitude of $S_{PDH,0}$. Simplemindedly, one might expect the servo to “get stuck” at one of these minima and not acquire lock if this minimum gain is less than unity. For high noise frequencies, it is more difficult to get into the regime where $S_{PDH,0}$ falls since β must exceed 1. It is much more likely with low frequency technical noise. Therefore, we start by assuming large f_{fm} and f_n□$\Delta\nu$ for which Eq. (6) gives the important limit to the servo gain. The other cases can be worked out similarly and one arrives at conclusions that are not very different.

We consider a servo control system that has a pole at DC and a zero at the cavity linewidth $\Delta\nu$. In this way, the system loop gain for small signals is $G=f_1/f_n$ with a phase lag of $\pi/2$, where f_1 is the unity gain frequency. For large amplitude noise f_{fm}, the gain is reduced by the ratio of Eq. 6 to Eq. 4 to:

$$G = 2\left(\frac{\Delta\nu}{f_{fm}}\right)^2 \frac{f_1}{f_n} \tag{7}$$

This gain is greater than one if $f_{fm}<(2f_1/f_n)^{1/2}\Delta\nu$. Treating the lock capture recursively, as this gain attacks the noise, the $S_{PDH,0}$ increases, producing more gain so that it may eventually reduce a large fm noise to a small fm amplitude.

Experimentally, we demonstrate that our servo can acquire lock when our laser has large fm noise. We can acquire lock over the full range of f_n with $f_{fm}\approx 1.5$ MHz and a loop unity gain frequency of $f_1=2$ MHz. We add noise either using AOM 1 or, for high f_n, by modulating the diode-laser current of Laser 1. For $f_n=2$kHz, Eq. 7 gives a loop gain of only G=0.01. Nonetheless, the servo acquires lock!

For very large fm, this simple linear model does not accurately describe the gain at f_n, even though Eq. 2 very accurately describes our measured $S_{PDH,0}$. It is important to note that the servo gain at higher harmonics of Ω contribute to the gain at Ω because it will change the relative intensity of all the noise sidebands that contribute to $S_{PDH,0}$ at frequency Ω. This effect is not included in this simple model. More work is needed to understand the non-linear dynamics of the lock acquisition transient in this regime. However, in the context of lasers for optical clocks, this question is not so urgent because one normally tries to insure that the laser has minimal technical noise at all frequencies.

VI. Summary

We have demonstrated that a noisy laser can be locked directly to a high Q cavity, even for large v_ω. Large frequency noise distorts the Pound-Drever-Hall error signal but the magnitude at the noise frequency falls slowly for large fm noise and its phase is bounded near zero. Experimentally, the lock capture range is much larger than the range given by the gain at the frequency of the noise. For large fm noise, higher harmonics of the noise also contribute to the gain at fundamental frequency. Therefore, analog servos can be used to acquire lock directly to a high Q cavity or to a pre-stabilization cavity with a very high finesse. A pre-stabilization cavity with $\Delta\nu=3$ kHz more strongly filters the frequency and intensity noise of the transmitted light than a cavity with $\Delta\nu=300$ kHz. Thus the cavity transmission, and possibly thermal considerations, not the laser linewidth, limits the finesse. Current mirrors offer a finesse>10^5 with 50% cavity transmission.

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