Locking lasers with large FM noise to high-Q cavities

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We demonstrate that a laser can be directly locked to a cavity when the laser linewidth is much greater than the cavity linewidth. We lock an external-cavity diode laser with more than 1 MHz of added frequency noise to a 3.5 kHz wide cavity resonance. Our analog servo acquires lock even though the laser frequency sweeps through the cavity resonance in less than the cavity buildup time. Our theoretical analysis fully describes our measurements and explains why lock can be acquired. © 2005 Optical Society of America

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Optical-frequency atomic clocks and experiments to detect gravitational radiation rely on precisely matching a laser's frequency to the resonant frequency of a passive optical cavity.[1,2] Passive cavities currently have quality factors greater than 10^{11}, and lasers locked to such cavities generate the most-stable frequencies.[3] When locking is to a narrow cavity the free-running laser linewidth may be much greater than the cavity linewidth, so the laser frequency sweeps through the cavity resonance in less time than it takes for light to build up in the cavity.[4–6] This leads to severe distortions of the error signal that is used to control the laser frequency, suggesting that a conventional servo cannot acquire lock.[6] This difficulty has been avoided by first locking to a prestabilization cavity whose linewidth is slightly smaller than the laser linewidth.[3,6] Here we show that error signal distortions in fact do not inhibit lock acquisition for narrow cavities and analog servos. Thus prestabilization cavities may be unnecessary for some experiments and, for others, can have a higher Q to better attenuate the frequency and amplitude noise of the transmitted light.

We lock an external-cavity diode laser to a cavity with a linewidth of 3.5 kHz, using a conventional servo. A series of experiments has locked diode lasers to successively narrower cavity resonances, as narrow as 9 kHz.[7,8] As cavity linewidths become narrower, the distortion threshold goes down quadratically. Our cavity resonance is almost a factor of 3 narrower than the previous best, leading to a distortion threshold that is seven times lower for the same laser noise.[5] Here we intentionally added large laser frequency modulation (>1 MHz) so we can clearly assess lock acquisition in this regime. Previous research for gravity wave detectors analyzed the linear motion of mirrors in the time domain by solving differential equations.[4–6] To correct for the large velocities of suspended interferometer mirrors, a real-time (digital) analysis of the error signal distortions was used to acquire lock.[2,4,5] Here we analyze this regime experimentally and theoretically in the frequency domain. Our analytic solution shows that, even for large distortion, the error signal is appropriate for a conventional analog servo to acquire lock. We first describe our calculation and measurement of the error signal and show that they agree well. We then experimentally and theoretically analyze the magnitude and phase of the error signal at the frequency of the noise.

We lock lasers to a cavity with the widely used Pound–Drever–Hall (PDH) technique.[9] It provides a high signal-to-noise ratio and a large bandwidth. To analyze the lock acquisition, we treat our real laser, shown in Fig. 1, as an ideal laser to which frequency noise is added.[11] We decompose the laser frequency noise into its Fourier components at frequency Ω. In Fig. 1, the 780 nm light from laser 1, after passing through a 2 m optical fiber, acquires frequency sidebands in an electro-optic phase modulator, EOM1. Some of the light may be resonant with the cavity, and the incident light reflected from the cavity interferes with the light coming out of the cavity.[9] The electric field at photodiode PD1 is $E=\Sigma_{m,n}(\delta+m\omega_{\mathrm{EO}}+n\Omega)J_m(\beta')J_n(\beta)\exp[-i(\omega+m\omega_{\mathrm{EO}}+n\Omega)t]/(\delta+m\omega_{\mathrm{EO}}+n\Omega+i\Delta\omega)$, where δ is the detuning from the cavity resonance; $\Omega=2\pi f_n$ and β describe laser frequency noise $f(t)=f_{\mathrm{FM}}\cos(\Omega t)$, with $\beta=f_{\mathrm{FM}}/f_n$; the cavity linewidth is $\Delta\omega=2\pi\Delta\nu$; and ω_{EO} and β' are the phase-modulation frequency and depth of EOM1 with $\omega_{\mathrm{EO}}\gg\Delta\omega$. We take β' to be small, the mirrors to be highly reflective, and all frequencies to be much less than the free spectral range.[10]

The PDH error signal S_{PDH} comes from mixing the photodiode current, proportional to E^*E, with $\sin(\omega_{\mathrm{EO}}t)$ and then low-pass filtering.[9] With no fre-

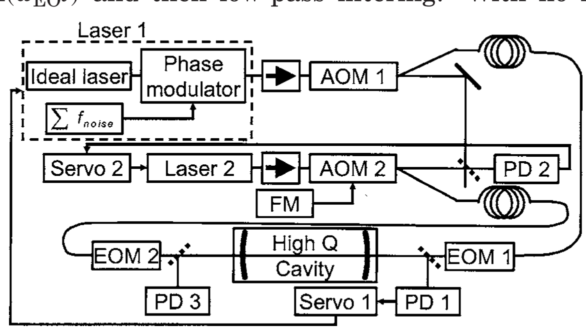

Fig. 1. We treat the real laser 1 as an ideal laser with added phase noise. We lock it to a high-finesse cavity after it passes through an optical isolator, acousto-optic modulator AOM1, a 2 m optical fiber, and an electro-optic modulator. The light reflected from the cavity is detected on photodiode PD 1. Laser 2 is phase locked to laser 1 via PD 2 and scanned over the cavity resonance with AOM 2. EOM 1 and EOM 2 operate at 56 and 72 MHz, and servos 1 and 2 have bandwidths of 2 and 4 MHz, respectively.

quency noise ($\beta=0$), the zero-frequency (dc) signal is $S_{\mathrm{PDH}}=\beta'\,\delta\Delta\omega/(\delta^2+\Delta\omega^2)$. In the presence of noise, we take the laser frequency to be bounded around and centered on the cavity resonance ($\delta=0$), yielding

$$S_{\mathrm{PDH}}=2\beta'\sum_{p,k=0}^{\infty}J_{k-p}(\beta)J_{k+p+1}(\beta)(2k+1)\Delta\omega\Omega \times\frac{[(k-p)(k+p+1)\Omega^2+\Delta\omega^2]\cos[(2p+1)\Omega t]+(2p+1)\Delta\omega\Omega\sin[(2p+1)\Omega t]}{[(k-p)^2\Omega^2+\Delta\omega^2][(k+p+1)^2\Omega^2+\Delta\omega^2]}. \tag{1}$$

Equation (1) gives the general S_{PDH} for high and low f_n and large and small FM noise depths.

After locking laser 1 to our cavity, we measure S_{PDH} with laser 2 by scanning it over an adjacent cavity mode. We remove the frequency noise of laser 2 by phase locking it to laser 1 with a frequency offset (240 MHz), using servo 2 (Fig. 1). We scan laser 2 over a cavity resonance by modulating the frequency of an acousto-optic modulator, AOM 2, using a direct digital synthesizer. AOMs 1 and 2 operate near 112 MHz such that the laser frequencies at the cavity are separated by one free spectral range of 464.59 MHz. Our cavity's linewidth is $\Delta\nu=3.46$ kHz.

We modulate the frequency of AOM 2 at frequencies $f_n=\Omega/2\pi=0.8$, 4, and 10 kHz, with FM depths $f_{\mathrm{FM}}<2$ MHz. In this way we measure S_{PDH} for sinusoidal noise in laser 2. In Fig. 2 we show S_{PDH} for $f_n=0.8$, 10 kHz and several noise depths f_{FM}. The measured (solid) and calculated [from Eq. (1), dashed] curves agree well.) For all S_{PDH} we use a single factor for the photodiode and amplifier gains, a single time offset for each f_n and a dc offset for each curve.

The data in Fig. 2 span seven regions of the response of S_{PDH}, which we show in Fig. 3. These seven regions show five distinct behaviors, which we now discuss, with smooth transitions among all regions. Two of the behaviors are in the well-known operating range when the laser is locked.[9] In regions 1_L and 1_H the noise is small ($\beta<1$) and Eq. (1) simplifies to

$$S_{PDH}\approx\beta' f_{\mathrm{FM}}\frac{\Delta\nu\cos(\Omega t)+f_n\sin(\Omega t)}{f_n^{\,2}+\Delta\nu^2}. \tag{2}$$

In Figs. 2(a) and 2(b) the measured S_{PDH} is sinusoidal for $f_{\mathrm{FM}}=0.8$ and 10 kHz with a phase lag of $\tan^{-1}(f_n/\Delta\nu)$.

In regions 3_{dc} and 2_L, any low-frequency noise can have a large phase-modulation index β because $\beta=f_{\mathrm{FM}}/f_n$. For $f_{\mathrm{FM}}=3.2$ kHz $\approx\Delta\nu$ in region 2_L, some distortion occurs in Fig. 2(a) because the dc response $S_{\mathrm{PDH}}=\beta'\,\delta\Delta\omega/(\delta^2+\Delta\omega^2)$ is not linear in δ as the frequency scans back and forth over $\pm f_{\mathrm{FM}}$. The distortion increases as f_{FM} (and β) grows, moving from 2_L to 3_{dc}.

In region 3_L the low-frequency response of S_{PDH} is more interesting when the FM depth is so large that the laser frequency changes by more than the cavity linewidth in a cavity lifetime.

When frequency velocity $v_\omega=\dot{\omega}/\Delta\omega^2\gg 1$, the distortions are large.[4–6] In Fig. 2(a) $v_\omega=0.85$ for $f_{\mathrm{FM}}=12.8$ kHz shows the onset of a fast beat between the sweeping incident light and the field stored in the cavity.[4–6] The distortions increase as v_ω goes to $v_\omega>50$ for $f_{\mathrm{FM}}=819.2$ kHz.

In region 3_H, the distortion is also large for large f_{FM} and high f_n in Fig. 2(b). For f_{FM} larger than those in Fig. 2(b), the distortions are similar to those for $f_n<\Delta\nu$ and $v_\omega>1$. However, $f_n>\Delta\nu$ means that the

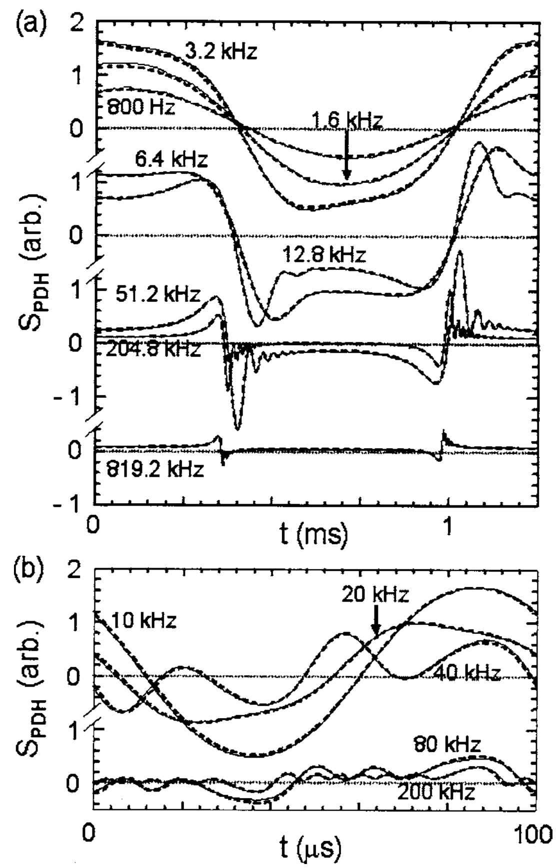

Fig. 2. PDH error signal S_{PDH} for a cavity linewidth of 3.46 kHz and noise at (a) $f_n=0.8$ kHz with depths 0.8, 1.6, 3.2, 6.4, 12.8, 51.2, 204.8, and 819.2 kHz and (b) $f_n=10$ kHz and noise depths 10, 20, 40, 80, and 200 kHz. Experiment (solid) and theory (dashed) agree well. The theory curves include the 16.34 kHz pole of the low-pass filter. For (a) $f_{\mathrm{FM}}=12.8$ kHz corresponds to $v_\omega=0.85$, and v_ω increases linearly with f_{FM}.

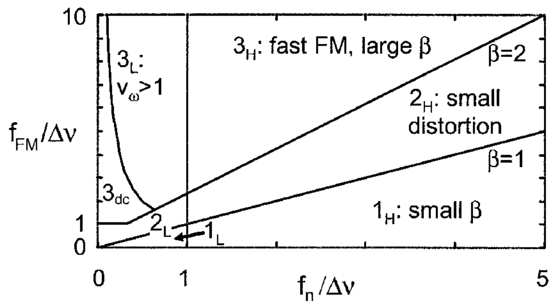

Fig. 3. Regions of the response of Pound–Drever–Hall error signal S_{PDH} as a function of noise frequency f_n and noise depth f_{FM}, scaled by cavity linewidth $\Delta\nu$.

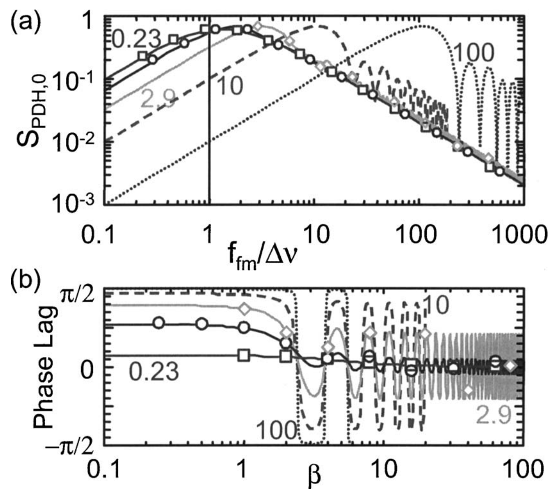

Fig. 4. (a) Magnitude and (b) phase of S_{PDH} at noise frequency $\Omega=2\pi f_n$ as a function of noise depth f_{FM} and $\beta=f_{\mathrm{FM}}/f_n$ for $f_n/\Delta\nu=0.23$, 1.15, 2.9, 10, 100. Data points are squares (circles, diamonds) for $f_n/\Delta\nu=0.23$ (1.15, 2.9). The response falls for large FM depths at all f_n. For $f_n>\Delta\nu$, the phase oscillates between a phase lag and a phase lead for $\beta>2$.

laser frequency returns to the cavity resonance in less than a lifetime, so v_ω is not meaningful.

Given the severe distortions for large v_ω or β in Fig 2, does S_{PDH} have a magnitude and a phase that will allow a servo to acquire lock? To answer this question we analyze the magnitude and phase of $S_{\mathrm{PDH},0}$, the Fourier component of S_{PDH} at noise frequency f_n; this is the $p=0$ term in Eq. (1). In Fig. 4 we show the magnitude and phase of S_{PHD} versus f_{FM} or β for several f_n. There are four distinct behaviors for small and large f_n and f_{FM}. For small f_n and f_{FM}, $S_{\mathrm{PDH},0}$ is given by Eq. (2). It increases linearly with f_{FM} and has a small phase shift. For small f_n and large f_{FM}, $S_{\mathrm{PDH},0}$ decreases as $1/f_{\mathrm{FM}}$, and the phase shift goes to 0 for large β. Here, Eq. (1) reduces to $S_{\mathrm{PDH},0}\approx 2\beta'\Delta\nu\cos(\Omega t)/f_{\mathrm{FM}}$, where the sidebands near the turning points of the frequency sweep dominate. Therefore, $S_{\mathrm{PDH},0}$ is entirely independent of v_ω. Because the magnitude falls slowly for large f_{FM} and the phase lag is small for all f_{FM}, a servo can use $S_{\mathrm{PDH},0}$ to acquire lock when $f_n<\Delta\nu$.

For large f_{N}, there are three distinct behaviors. For small β, $S_{\mathrm{PDH},0}$ is again given by Eq. (2) and is proportional to the amplitude of the FM noise. For most large β, the leading $p=0$ term in Eq. (1) is due to the carrier and first-order sidebands, $S_{\mathrm{PDH},0}\approx 2\beta' J_0(\beta)\sin(\Omega t)$. It decreases as $1/f_{\mathrm{FM}}(1/\beta)$ and changes sign for every increase of β by $\pi/2$. Therefore the phase of $S_{\mathrm{PDH},0}$ in Fig. 4(b) regularly switches from the usual phase lag of nearly $\pi/2$ to a phase lead of nearly $\pi/2$. For large β, the minimum magnitude of $S_{\mathrm{PDH},0}$ in Fig. 4(a) is that given above for small f_n and large f_{FM}. Because the phase lag never exceeds $\pm\pi/2$, a servo can capture lock.

To demonstrate that a laser with large frequency noise can acquire a lock to the cavity, we add noise with $f_{\mathrm{FM}}\approx 1.5$ MHz, much greater than our free-running laser 1 linewidth, over the full interesting range of f_n. Our capture range of $f_{\mathrm{FM}}\approx 1.5$ MHz is larger than a prediction from Fig. 4.[11] We note that odd harmonics of Ω contribute to the gain at Ω. This nonlinearity is not included in Fig. 4.

We have demonstrated that a noisy laser can be locked directly to a high-Q cavity, even for large v_ω. Large FM noise distorts the Pound–Drever–Hall error signal, but the response at the noise frequency falls slowly for large FM noise, and its phase is bounded near 0. Therefore a conventional servo can acquire lock directly to a high-Q cavity. A high-Q prestabilization cavity strongly filters the frequency and intensity noise of the transmitted light. Thus the cavity transmission, and possibly thermal effects limits the finesse. Current mirrors offer a finesse of 10^5 with 50% cavity transmission.

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