USEFUL TERMINOLOGY FOR UNDERSTANDING DIFFERENTIAL EQUATIONS

Notation for Derivatives:

The most common notation methods are **Lagrange notation** (aka prime notation), **Newton notation** (aka dot notation), and **Leibniz's notation** (aka dy/dx notation).

Ex 1:

Lagrange Notation: $y''(x) = 0$

Newton Notation: $\ddot{y} = 0$

Leibniz Notation: $\frac{d^2y}{dx^2} = 0$

The example above shows three different ways to write the second derivative of y is equal to zero. Note that Leibniz notation is the notation used for the rest of the reference sheet.

Independent Variable:

The variable in an equation that can be freely chosen and does not depend on another variable.

Dependent Variable:

The variable that depends on the value of at least one independent variable.

Ex 2:

$$\frac{dy}{dx} = x + 2$$

The variable y is the dependent variable. Variable x is the independent variable. y is a function of x and can be denoted y = y(x). Note how y is in the numerator and x is in the denominator of the derivative.

Differential Equation:

An equation that contains an unknown function and its derivatives.

Ex 3:

$$\frac{dy}{dx} + y = 0$$

The example contains the dependent variable y and its derivative.
Again remember that y is a function of x and can be denoted y = y(x).

Ordinary Differntial Equation (ODE):

A differential equation that is written in terms of one independent variable.

Ex 4:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x$$

The example above is written in terms of independent variable x, where y is a function of x. All examples given so far have been ODEs.

Partial Differential Equation (PDE):

In contrast to an ODE, a partial differential equation is a differential equation written in terms of more than one independent variable.

Ex 5:

$$\frac{dy}{dx} + \frac{dy}{dv} = x$$

$$y = f(x,v)$$

The example above is written in terms of independent variables x and v. The dependent variable is y, where y is a function of both x and v.

Order:

The value of the highest derivative of an ODE. If given a system of equations, the order of the system is the sum of the order of each equation.

Ex 6:

$$\frac{d^2y}{dx^2} = x$$

The highest derivative in the example is two. Therefore, it is a second-order equation. Examples 1 and 4 also show second order equations. Examples 2,3 and 5 are first order equations. The term 'Higher order' refers to an order of three or more.

Separable:

When the variables of an ODE can be rearranged on to opposite sides of the equal sign.

Ex 7:

$$\frac{dy}{dx} = x + xy$$

$$\frac{dy}{dx} = x(1 + y)$$

$$\frac{dy}{(1 + y)} = xdx$$

The example ODE equation was first factored into x and (1+y). The term (1+y) was divided to the left hand side. The x variable was multiplied to the right hand side. Separating equations in this way allows for easy integration.

Linear:

An equation that forms a line when plotted. The dependent variable should always be to a power of 1 and should not be multiplied by another dependent variable term.

Ex 8:

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy \ = \ 0$$

Ex 9:

$$a\frac{d^2y}{dx^2}\frac{dy}{dx} + b\frac{dy}{dx} + cy^2 \ = \ 0$$

Example 8 is the form of a second-order linear equation with coefficients a,b, and c. Example 9 is a non-linear second-order equation with the same coefficients. Note why the equations are different.

Homogeneous:

A linear equation that is equal to zero when only the dependent variable terms are on the left-hand side of the equal sign.

Ex 10:

$$\frac{dy}{dx} + y = 0$$

The example above is homogenous. Examples 1,3, and 8 are also homogeneous. Examples 2,4-7, and 9 are not homogenous.